RH: Phylogenetic analysis of host repertoire evolution

Bayesian inference of ancestral host-parasite interactions under a phylogenetic model of host repertoire evolution

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Abstract.— Intimate ecological interactions, such as those between parasites and their hosts, may persist over long time spans, coupling the evolutionary histories of the lineages involved. Most methods that 2 reconstruct the coevolutionary history of such associations make the simplifying assumption that parasites 3 have a single host. Many methods also focus on congruence between host and parasite phylogenies, using 4 cospeciation as the null model. However, there is an increasing body of evidence suggesting that the host 5 ranges of parasites are more complex: that host ranges often include more than one host and evolve via 6 gains and losses of hosts rather than through cospeciation alone. Here, we develop a Bayesian approach for 7 inferring coevolutionary history based on a model accommodating these complexities. Specifically, a 8 parasite is assumed to have a host repertoire, which includes both potential hosts and one or more actual 9 hosts. Over time, potential hosts can be added or lost, and potential hosts can develop into actual hosts or 10 vice versa. Thus, host colonization is modeled as a two-step process, which may potentially be influenced 11 by host relatedness or host traits. We first explore the statistical behavior of our model by simulating 12 evolution of host-parasite interactions under a range of parameters. We then use our approach, 13 implemented in the program RevBayes, to infer the coevolutionary history between 34 Nymphalini 14 butterfly species and 25 angiosperm families. 15

16 (Keywords: ancestral hosts, coevolution, herbivorous insects, probabilistic modeling.)

Extant ecological interactions, such as those between parasites and hosts, are often the 17 result of a long history of coevolution between the involved lineages (Elton 1946; Klassen 1992). 18 Specialization is predominant among parasites (including parasitic herbivorous insects; Forister 19 et al. 2015), but host associations are not static: they continuously evolve over time via gains and 20 losses of hosts (Janz and Nylin 2008; Nylin et al. 2018). The colonization of new hosts and loss of 21 old hosts not only shape the evolutionary trajectories of the interacting lineages, but can also 22 have large effects at ecological timescales (Nosil 2002; Calatayud et al. 2016). These effects are 23 evident, for example, with emerging infectious diseases and zoonotic diseases (Acha and Szyfres 24 2003), which involve colonization of new hosts within and among groups of domesticated species 25 (Subbarao et al. 1998), wildlife (Fisher et al. 2009), and humans (Hahn et al. 2000). Unraveling 26 the processes underlying changes in species associations is thus key to understanding evolutionary 27 and ecological phenomena at various timescales, such as the emergence of infectious diseases, 28 community assembly, and parasite diversification (Hoberg and Brooks 2015). 29

Many methods developed to study historical associations focus on congruence between 30 host and parasite phylogenies (Brooks 1979; Huelsenbeck et al. 1997; de Vienne et al. 2013). Such 31 methods largely fall into two main classes of cophylogenetic approaches: (1) topology- and 32 distance-based methods, which estimate the congruence between two phylogenies (Legendre et al. 33 2002), and (2) event-based methods, which map the parasite phylogeny onto the host phylogeny 34 using evolutionary events (Ronquist 2003). Typically, cospeciation is the null hypothesis in these 35 methods, where host shifts are invoked only to explain deviations from cospeciation (de Vienne 36 et al. 2013). Moreover, most of these methods do not allow ancestral parasites to be associated 37 with more than one host lineage, thus failing to account for a potentially important driver of 38 parasite diversification (Janz and Nylin 2008). 39

An alternative approach to studying coevolving host-parasite associations is to perform ancestral state reconstructions of individual host taxa onto the parasite phylogeny and combine the ancestral host states *a posteriori* into inferred host ranges (e.g. Nylin et al. 2014). Even though this approach allows ancestral parasites to have multiple hosts, it assumes that the

associations between the parasite and each host evolve independently. This has a number of
serious drawbacks. For instance, ancestral parasites may be inferred to have an unrealistically
high number of hosts, or no host at all. Furthermore, the more narrowly circumscribed the host
taxa are, the more likely it is that ancestral parasite lineages are reconstructed as having no
hosts. In addition, the independence assumption causes the phylogenetic relationships among
hosts to be ignored, meaning that the model assigns equal rates to all colonizations of new hosts
regardless of how closely related the new host is to the current hosts being used by the parasite.

A desirable model of host usage should therefore allow parasites to have multiple hosts, 51 while also allowing for among-host (or context-dependent) effects to influence ancestral host use 52 estimates and gain and loss rates in whatever manner explains the biological data best. One 53 possible solution is to restate the problem of host-parasite co-evolution in terms of historical 54 biogeography. For instance, the Dispersal-Extirpation-Cladogenesis (DEC) model of Ree et al. 55 (2005) allows species ranges to stochastically evolve as a set of discrete areas over time through 56 area gain events (dispersal), area loss events (extirpation), and cladogenetic events (range 57 inheritance patterns that reflect speciational models). Although these methods are designed for 58 biogeographic inference, a similar approach is clearly suitable for more realistic modeling of 59 host-parasite coevolution dynamics, where colonization and loss of hosts (instead of discrete 60 areas) is modeled as a continuous-time Markov process (e.g. Hardy 2017). In biogeography, the 61 colonization of a new area or the disappearance from a previously occupied area is modeled as a 62 binary trait: the species is either present or absent in the area. While this binary view might be 63 simple but useful in biogeography, it may be too simplistic for use in the coevolution between 64 hosts and parasites. For instance, it is known that butterflies can utilize a range of plants that 65 they do not regularly feed on in the wild, and it has been suggested that these potential hosts 66 have played an important role in the evolution of host use in butterflies, by increasing the 67 variability in host use through time and across clades (Janz et al. 2016; Braga et al. 2018). This 68 hypothesis can only be directly tested, however, if we explicitly model the evolution of host use as 69 a two-step process, which cannot be done with the binary methods that are used today to study 70

⁷¹ host-parasite coevolution or biogeography.

Here, we propose a model where a parasite is assumed to have a *host repertoire*, defined as 72 the set of all potential and actual hosts for that parasite. In this model, the colonization of a new 73 host involves two steps: first, the parasite gains the ability to use the new host (it becomes a 74 potential host), and then starts actually using it in nature (it becomes an actual host). These two 75 steps can be interpreted as the inclusion of the new host into the fundamental and then into the 76 realized host repertoire of the parasite - analogous to fundamental and realized niche (Nylin et al. 77 2018; Larose et al. 2019). Similarly, the complete loss of a host from a parasite's realized 78 repertoire involves two steps. First, it changes from an actual to a potential host, and then it is 79 lost completely from the host repertoire. For example, if the geographic range of a host 80 contracted to become allopatric with respect to a parasite's geographic range, the host would 81 remain as part of the fundamental repertoire until the parasite completely lost the ability to use 82 the host, in which case the host would be lost from the repertoire. Even when in sympatry, the 83 evolution of a new defense mechanism by the host may prevent the parasite from using that host. 84 However, since host use is a complex and multidimensional trait, it is unlikely that a parasite 85 loses all the machinery necessary to use a host in one single event, and it may well retain some 86 ability to survive on the host. Thus, three host-parasite association states are necessary for such a 87 two-step model: the host is used (actual host), the parasite has some ability to use the host but 88 does not use it in nature (potential host), and the parasite cannot use the host (non-host). 89

In this paper, we develop a Bayesian approach to coevolutionary inference based on such a 90 model of host repertoire evolution, inspired by the previous work on similar biogeographic 91 inference problems by Landis et al. (2013). The basic binary biogeographic model, when applied 92 to coevolution, accommodates both multiple ancestral hosts and changes in host configurations 93 over time that correspond to evolutionary changes in host lineages or host traits. We extend this 94 model to also include a two-step host colonization process, such that the fundamental host 95 repertoire can persist over time and affect the evolution of the realized repertoire. We have 96 implemented the model in RevBayes (Höhna et al. 2016), allowing us to perform simulation as 97

well as Bayesian Markov chain Monte Carlo (MCMC) inference under the model. This Bayesian framework allows one to estimate the joint distribution of host gain and loss rates, the effect (if any) of phylogenetic distances among hosts upon host gain rates, and the historical sequences of evolving host repertoires among the parasites. Using simulations, we explore the statistical behavior of our approach, and demonstrate its empirical application with an analysis of the coevolution between Nymphalini butterflies and their angiosperm hosts.

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Methods

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Model description

We are interested in modeling the evolution of ecological interactions between M extant parasite taxa and N host taxa, where each parasite uses one or more hosts. Rooted and time-calibrated phylogenetic trees describe the evolutionary relationships among the M parasite taxa and among the N host taxa. In this study, the trees are considered to be known without error. In principle, it would be straightforward for the model to accommodate phylogenetic uncertainty in the host or parasite trees but MCMC inference may prove challenging under such conditions.

Each parasite taxon has a host repertoire, which is represented by a vector of length N113 that contains the information about which hosts the given parasite uses. The interaction between 114 the *m*-th parasite and the *n*-th host is denoted $x_{m,n}$. At any given time, each host taxon can 115 assume one of three states with respect to a parasite lineage: $x_{m,n}$ is equal to 0 (non-host), 1 116 (potential host), or 2 (actual host). Criteria for how to code non-host, potential host, and actual 117 host states will depend on the host-parasite system under study; below, we provide criteria for 118 our Nymphalini dataset that may act as guidelines. We allow all host repertoires in which the 119 parasite has at least one actual host. Thus, the state space, S, includes $3^N - 2^N$ host repertoires 120 for N hosts. 121

Here we define the transition from state 0 to state 1 as the gain of the ability to use the 122 host, and the transition from state 1 to state 2 as the time when the parasite actually starts to 123 use the host in nature. If we assume that gains and losses of hosts occur according to a 124 continuous-time Markov chain, the probability of a given history of association between a parasite 125 clade and their hosts can be easily calculated (Ree and Smith 2008). This calculation is based on a 126 matrix, Q, containing the instantaneous rates of change between all pairs of host repertoires, and 127 thus describing the Markov chain. Based on the \mathbf{Q} matrix, it is possible to calculate the transition 128 probability of the observed host repertoires at the tips of the parasite tree by marginalizing over 129 the infinite number of histories that could produce the observed host repertoires. Unfortunately, 130 computing these transition probabilities becomes intractable as the number of host repertoire 131 configurations, S, grows large. Modeling host repertoire evolution for host repertoire size N = 7132 requires an $S \times S$ rate matrix defined for $S = 3^7 - 2^7 = 2059$, causing **Q** to be too large for 133 efficient inference. In order to handle large host repertoires, we numerically integrate over possible 134 histories using data augmentation and MCMC rather than analytically computing the 135 probabilities using matrix exponentiation. This data augmentation approach has been used to 136 model sequence evolution for protein-coding genes (Robinson 2003) and historical biogeography 137 (Landis et al. 2013; Quintero and Landis 2019), suggesting the framework may be useful to model 138 host-parasite interactions as well. In this study, we assume that both daughter lineages identically 139 inherits their host repertoires from their immediate ancestor at the time of cladogenesis. 140

We define a model where the gain of a host (both $0 \rightarrow 1$ and $1 \rightarrow 2$) depends on the 141 phylogenetic distance between the available hosts and those currently used by a lineage. Figure 1 142 schematically illustrates the evolutionary dynamics of the model using M = 4 parasite species and 143 N = 5 host species, while assuming that host gain rates are independent (Fig. 1a,c) or dependent 144 (Fig. 1b,d) of phylogenetic distances among hosts. To formalize these dynamics, let $q_{\mathbf{y},\mathbf{z}}^{(a)}$ be the 145 rate of change from host repertoire y to repertoire z by changing the state of host a. Also, let λ_{ij} 146 be the rate at which an individual host changes from state i to state j, and $\eta(\mathbf{y}, a, \beta)$ be a 147 phylogenetic-distance rate modifier. The phylogenetic-distance rate modifier function, η , rescales 148

the base rate of host gain to allow new hosts that are closely related to the parasite's current
hosts to be colonized at higher rates than distantly related hosts. We define the instantaneous
rate of change as

$$q_{\mathbf{y},\mathbf{z}}^{(a)} = \begin{cases} \lambda_{10}, & \text{if potential host loss } (y_a = 1 \text{ and } z_a = 0) \\ \lambda_{01}\eta_1(\mathbf{y}, a, \beta) & \text{if potential host gain } (y_a = 0 \text{ and } z_a = 1) \\ \lambda_{21}, & \text{if actual host loss } (y_a = 2 \text{ and } z_a = 1) \\ \lambda_{12}\eta_2(\mathbf{y}, a, \beta) & \text{if actual host gain } (y_a = 1 \text{ and } z_a = 2) \\ 0, & \text{if direct transition between states } 0 \text{ and } 2 (|y_a - z_a| > 1) \\ 0, & \text{if } \mathbf{y} \text{ and } \mathbf{z} \text{ differ at more than one host} \\ 0 & \text{if } \mathbf{z} \text{ does not contain at least one actual host} \end{cases}$$

¹⁵² and the phylogenetic-distance rate modifier function as

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$$\eta(\mathbf{y}, a, \beta) = e^{-\beta d/\overline{d}},\tag{1}$$

where β controls the effect of d, the average pairwise phylogenetic distance between the new host, 153 a, and the hosts currently occupied in y; and \overline{d} is the average phylogenetic distance between all 154 pairs of hosts. Pairwise phylogenetic distance is defined as the sum of branch lengths separating 155 two leaf nodes. The difference between η_1 and η_2 is that in the first, pairwise distances are 156 calculated between the new host and all potential and actual hosts, while in the second only 157 actual hosts are included. This allows for a model formulation where the effect of host distances 158 on λ_{01} and on λ_{12} are independent, while still allowing a formulation where they are equal. If 159 $\beta = 0$, the gain rate of host a is equal to the unmodified gain rate, λ_{01} or λ_{12} . If $\beta > 0$, the gain 160 rate of phylogenetically close hosts is higher than distant hosts. 161

¹⁶² We fit this model using the Bayesian data augmentation strategy described in Landis

tes al. (2013). The method estimates the joint posterior probability of model parameters,

 $\theta = (\mu, \lambda, \beta)$, and data-augmented evolutionary histories, X_{aug} , conditional on the observed host 164 repertoire data, $X_{\rm obs}$, and the parasite phylogeny, $\Psi_{\rm p}$, and the host phylogeny, $\Psi_{\rm h}$, using MCMC. 165 To sample values from the posterior, $P(X_{\text{aug}}, \theta \mid X_{\text{obs}}, \Psi_p, \Psi_h)$, new parameter values for μ , λ , and 166 β are proposed using standard Metropolis-Hastings proposals for updating simple parameters 167 (Hastings 1970). Analogously, our MCMC stochastically proposes and/or accepts new augmented 168 host repertoire histories using the Metropolis-Hastings algorithm. Augmented histories are 169 proposed using two types of MCMC moves: branch-specific moves and node-and-branch moves. 170 Branch-specific moves propose a new augmented history by sampling a branch from the 171 phylogeny uniformly at random, then proposing new histories for a subset of host-characters using 172 the rejection sampling method of Nielsen (2002) under the assumption that all host characters 173 evolved under mutual independence ($\beta = 0$); this assumption allows us to rapidly propose new 174 augmented histories. Although augmented histories are proposed assuming host characters evolve 175 independently, we compute the acceptance probability for the branch-specific move by considering 176 the full-featured model probability that allows for non-independent rates of character change 177 when calculating the Metropolis-Hastings ratio. Thus, the augmented histories are sampled in 178 proportion to their posterior probabilities under the full model. Node-and-branch moves involves 179 sampling new host repertoire states for a node sampled uniformly at random within the parasite 180 tree, along with the three branches incident to the node. Together, the branch-specific moves, the 181 node-and-branch moves, and the parameter moves allow MCMC to estimate the posterior 182 probability of combinations of host repertoire histories and evolutionary parameters. Further 183 details are provided in Landis et al. (2013). 184

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Model selection

When $\beta = 0$, the phylogenetic-distance dependent model, M_D becomes a mutual-independence model, M_0 , where the interaction between the parasite and each host evolves independently. These models are therefore nested ($M_0 \subseteq M_D$) and we can compute Bayes factors for model M_D



Figure 1: Host repertoire evolution along a hypothetical tree and resulting host-parasite interactions. Two examples of coevolutionary histories between four parasites and five hosts are shown to illustrate how the model works. Host repertoires evolve by gains $(0\rightarrow 1 \text{ and } 1\rightarrow 2$, blue arrows) and losses $(1\rightarrow 0 \text{ and } 2\rightarrow 1, \text{ red arrows})$. Coevolutionary histories in **a** and **b** produce the interactions in **c** and **d** respectively. In **c** and **d**, each column represents one host and each row represents the host repertoire of one parasite. High phylogenetic conservatism is produced when the rate of repertoire evolution, μ , is low and the effect of the phylogenetic distance between hosts, β , is high. Conversely, low phylogenetic conservatism is produced when μ is high and β is low.

over model M_0 using the Savage-Dickey ratio (Verdinelli and Wasserman 1995; Suchard et al. 2001), defined as

$$B_{D,0} = \frac{P(\beta = 0 \mid M_D)}{P(\beta = 0 \mid \mathbf{x}_{obs}, M_D)}$$
(2)

where $P_D(\beta = 0 \mid M_D)$ is the prior probability and $P(\beta = 0 \mid \mathbf{x}_{obs}, M_D)$ is the posterior 191 probability, both defined in terms of the phylogenetic-distance dependent model, M_D , at the 192 restriction point $\beta = 0$ where M_D and M_0 are equivalent. While we could directly compute the 193 prior probability of $\beta = 0$, we approximated the posterior at $\beta = 0$ using a kernel density 194 estimator with a gamma function, which only takes positive values, and a bandwidth of 0.02. To 195 interpret if and how Bayes factors favored the phylogenetic-distance dependent model, M_D , we 196 followed the guidelines of Jeffreys (1961): model M_0 is favored for Bayes factors with values less 197 than 1, insubstantial support is awarded to model M_D for values between 1 and 3, substantial 198 support for values between 3 and 10, strong support for values between 10 and 30, very strong 199 support for values between 30 and 100, and decisive support for values greater than 100. 200

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Data analysis

Simulation study.— We simulated 50 datasets for each of nine combinations of values for the rate 202 of host-repertoire evolution, μ (0.01, 0.04, and 0.1), and values of β (0, 1, and 4). These 203 parameter combinations produce datasets with varying degrees of phylogenetic conservatism for 204 both parasites and hosts (Fig. 2). Each dataset contained 34 insects and 25 hosts, and was 205 produced by simulating host repertoire evolution in the parasite tree used in the empirical study 206 (see below). Host gain and loss rates were chosen to resemble the rates inferred from the 207 empirical analysis. This simulation was designed to assess our statistical power to detect the 208 effect of phylogenetic distance among hosts upon host gain rates given the size of our empirical 209 dataset and the type of variation we expected it to contain. 210

We ran independent MCMC analyses for each set of 50 datasets, under the phylogenetic-distance dependent model. We then quantified how well the posterior probabilities



Figure 2: Simulated datasets for nine parameter combinations. Interactions between Nymphalini butterflies and their host plants for one of 50 simulations with each parameter combination. In each of the nine datasets, each column represents one host in the repertoire and each row shows the host repertoire of one butterfly species. When phylogenetic conservatism in host-parasite interactions is low for both hosts and parasites, the interactions are more randomly spread (matrix at bottom-left corner). As phylogenetic conservatism among parasites increases, host repertoires (rows) become more similar (upper matrices). When there is phylogenetic conservatism among hosts, host repertoires include more closely-related hosts (neighbouring columns; matrices to the right)

²¹³ of coevolutionary histories correspond to the true history known from each simulation.

Specifically, we first computed the posterior probability of interaction between each host and each internal node in the butterfly tree, for states 1 and 2 separately. Then, we calculated the sum of squared differences between each posterior probability ($0 \le P \le 1$) and the corresponding truth for that simulation (1, if the host was on the given state in the simulated dataset; 0, if not). This error term increases as the inferred ancestral host repertoires become less accurate.

Empirical study.— In order to validate our method, we compiled data from the literature for 219 butterflies from the tribe Nymphalini (Nymphalidae) and their host plants (see Supplementary 220 Information for reference list). We chose this butterfly clade because we expect that a large 221 fraction of the real potential hosts are known, as there has been systematic experimental studies 222 of larval feeding ability. The dataset included 34 butterflies species and plants from 16 223 angiosperm families (Figs. S1 and S2). For each butterfly species, host plants commonly used in 224 nature were coded as 'actual hosts' and plants never used were coded as 'non-hosts'. Plants that 225 are not commonly used in nature, but for which there is strong evidence (field observation or 226 experiment) that the larvae can feed upon them, were coded as 'potential hosts'. 227

Because we lack the information on potential hosts for most host-parasite systems (i.e. 228 hosts are usually only classified as hosts or non-hosts), we tested whether our model is able to 229 recover the same parameter estimates and coevolutionary histories when all the potential hosts 230 are coded as non-hosts. For that, we ran the same analysis as for the full dataset, but first 231 removed all the 1s from the empirical dataset. Then we compared the posterior probabilities 232 inferred from the two datasets. To assess the similarities between the coevolutionary histories 233 inferred using the different datasets, we calculated summary statistics for the absolute difference 234 in probability of each interaction between hosts and internal nodes in the butterfly tree. 235

For both the simulation and empirical studies we used the phylogenetic relationships between butterfly species in the Nymphalini tribe as proposed by Chazot (unpublished, Fig. S3) and the phylogenetic relationships between angiosperm families proposed by Magallón et al. (2015). Although our framework allows the inclusion of a large number of hosts in the same

analysis, computational time increases significantly with the size of the host repertoire. We 240 therefore chose to include 25 hosts, which allows the inclusion of all host lineages used by any of 241 the butterflies. To ensure the inclusion of all plant lineages that might have been used as hosts in 242 the past, we pruned the angiosperm phylogenetic tree so that all 16 families in the dataset were 243 included, and the remaining branches were collapsed to more ancestral nodes until only 25 tips 244 were left. We then pruned all the branches leading up to the tips to the time of origin of the 245 butterfly clade (approx. 22 Ma), and this pruned tree was then used to calculate phylogenetic 246 distances between hosts. To simplify the analysis, we hold the phylogenetic distances between 247 plant families constant, independent of geological time, even though the distances would be 248 expected to increase as evolution proceeds towards the recent. 249

We summarized inferred coevolutionary histories in two ways. First, we calculated the 250 posterior probability for fundamental and realized host repertoires at internal nodes of the 25 Nymphalini phylogeny based on the frequency with which states 1 and 2 were sampled for each 252 host during MCMC. Second, in order to reduce the dimensionality of the host repertoire and 253 facilitate visualization of ancestral state reconstructions, we assigned hosts to modules based on 254 extant butterfly-plant interactions (Fig. S2). Modules are groups of plants and butterflies that 255 interact more with each other than with other taxa, thus host plants are assigned to the same 256 module when they are used by the same butterflies. To identify the modules, we used a simulated 257 annealing algorithm that maximizes the index of modularity. Specifically, we used Newman and 258 Girvans metric (Newman and Girvan 2004) modified for bipartite networks (Barber 2007) as 259 implemented in the software MODULAR (Marquitti et al. 2014). 260

Software configuration.— All analyses were performed in RevBayes (Höhna et al. 2016). For the simulated data, we ran two independent MCMC analyses for 10^5 cycles, sampling parameters and node histories every 50 cycles, and discarding the first 5×10^4 as burnin. For the empirical data, we ran five independent MCMC analyses, each set to run for 10^6 cycles, sampling every 50 cycles, and discarding the first 10^5 as burnin. To verify that MCMC analyses converged to the same posterior distribution, we applied the Gelman diagnostic (Gelman and Rubin 1992) provided

through the R package *coda* (Plummer et al. 2006). For both simulated and empirical datasets,

- we used the following priors: $\beta \sim \text{Exponential}(1), \mu \sim \text{Exponential}(10), \text{ and}$
- $\lambda \sim \text{Dirichlet}(1, 1, 1, 1)$. Analysis scripts and data files are available at
- 270 https://github.com/mpiresbr/host_repertoire. A RevBayes tutorial for the empirical
- analysis will be soon available at https://revbayes.github.io/tutorials#host_rep.

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RESULTS

Simulation study.—Posterior distributions of parameter values for the 9×100 MCMC analyses are
shown in Figure 3. Overall, the model was able to accurately recover the true simulation
parameters (true value within 95% highest posterior density, or HPD). However, accuracy
decreased with increasing rate of host repertoire evolution, possibly due to character saturation.

We performed model selection based on Bayes factors. Considering that the prior distribution is $\beta \sim Exponential(1)$, a high marginal posterior probability for $\beta = 0$ under M_D is necessary to result in a Bayes factor < 1 and thus selection of M_0 . For simulations with $\beta = 0$, the correct model, M_0 , was selected in more than 60% of the simulations, and most of the remaining simulations gave insubstantial support to M_D (Fig. 4). When $\beta = 1$, Bayes factors correctly selected M_D in the majority of cases, but strong support for M_D was only achieved in simulations with $\beta = 4$, particularly when the rate of evolution was highest ($\mu = 0.1$).

We then compared the true coevolutionary history of each simulation to the corresponding 284 posterior distribution of the sampled coevolutionary histories (Fig. 5). The estimation error, that 285 is, the sum of squared differences between estimated and true coevolutionary histories, was very 286 low when the rate of host-repertoire evolution was lowest ($\mu = 0.01$), but also when the 287 phylogenetic-distance power was highest ($\beta = 4$). This means that accuracy in the estimation of 288 coevolutionary history increases with phylogenetic conservatism on both the butterfly and the 289 plant trees. Overall, error was higher on the estimation of actual hosts (state 2) than potential 290 hosts (state 1). 291



Figure 3: Posterior densities of parameters in the simulation study. Panels **a** and **b** are faceted by true parameter values of μ and β , respectively. Fifty datasets were simulated for each combination of $\beta \in \{0, 1, 4\}$ and $\mu \in \{0.01, 0.04, 0.1\}$, while $\lambda_{01} = 0.03$, $\lambda_{10} = 0.6$, $\lambda_{12} = 0.27$, and $\lambda_{21} = 0.1$ were held constant. For each parameter combination, the posterior distributions of the two MCMC samples of the 50 datasets were combined. Means are represented by black dots, black vertical lines show the 95% HPD, and red horizontal lines mark the true parameter value used in the simulations. Y-axis in panel **a** is in log_{10} scale for better visualization.



Figure 4: Distribution of Bayes factors for the simulation study. Each column corresponds to the strength of support per 2×50 MCMC analyses.



Figure 5: Errors for inferred dispersal histories of simulation study. The sum of squared differences between the posterior probability ($0 \le P \le 1$) and the true history (P = 0 or 1) for each host and each internal node were computed per simulated dataset. Each violin plot shows the distribution of these sums for each batch of 50 simulated datasets. Means are represented by black dots, black vertical lines show the 95% CI. Values of phylogenetic-distance power (β) are shown in the x-axis, columns are separated by the host-repertoire evolution rate (μ), and each row shows the error on the inference of each character state, i.e. potential host (1) or actual host (2).

Empirical study.— The estimated mean rate of host repertoire evolution for Nymphalini was 292 $\mu = 0.025$, the mean phylogenetic-distance power was $\beta = 0.51$, and the mean gain/loss rates were 293 $\lambda_{01} = 0.012$, $\lambda_{10} = 0.6$, $\lambda_{12} = 0.27$, and $\lambda_{21} = 0.12$ (Fig. 6, blue). Our method recovered similar 294 parameter estimates for the empirical dataset when omitting the intermediate state at the tips – 295 i.e. coding all potential hosts (state 1) as non-hosts (state 0): $\mu = 0.031$, $\beta = 0.39$, $\lambda_{01} = 0.001$, 296 $\lambda_{10} = 0.71$, $\lambda_{12} = 0.28$, and $\lambda_{21} = 0.01$ (Fig. 6, orange). The posterior distributions from analyses 297 with and without the intermediate state at the tips diverged the most for the rate parameters 298 associated with the transition to the intermediate state, λ_{01} and λ_{21} . In both cases the transition 299 rate was underestimated when 1s were removed from the dataset. Bayes factors selected the 300 independence model, M_0 , for both the full dataset (BF = 0.43) and when 1s were removed from 301 tip states (BF = 0.40). 302



Figure 6: Marginal posterior densities for parameters in the Nymphalini-Angiosperms study for both the full dataset (3 states at tips) and the dataset omitting the intermediate state (2 states at tips). Grey lines corresponds to the priors $\beta \sim \text{Exponential}(1)$, $\mu \sim \text{Exponential}(10)$, and $\lambda \sim \text{Dirichlet}(1, 1, 1, 1)$.

Finally, we reconstructed the fundamental and realized host repertoires at internal nodes of the Nymphalini phylogeny based on the sampled histories during MCMC. Coevolutionary histories inferred using the datasets with and without potential hosts were very similar, with mean difference in interaction probability of 0.003. Thus, we only show the ancestral states

inferred from the full, three-state dataset (Figs. 7 and S4). To facilitate visualization of the
ancestral state reconstruction, we grouped the 16 parasitized host families into five modules, as
identified by the simulated annealing algorithm (Fig. S2). Nine families (representing three
modules) were inferred to be used by ancestral Nymphalini species with high probability.



Figure 7: Evolution of butterfly-plant interactions through time. Ancestral state estimates (left) of host repertoire across the Nymphalini phylogeny are shown for interactions with more than 75% posterior probability. The x-axis under shows time before present in millions of years. Extant species interactions (right) between Nymphalini and their host plants are presented as a raster, where each square represents one interaction between a butterfly species and a host family. Colors represent different modules, i.e. groups of plants that are often hosts to the same butterflies at present time. Square size was used to differentiate between actual and potential hosts. Arrows indicate nodes shown in Fig. 8.

³¹¹ We found strong support for the association between the ancestor of all Nymphalini

³¹² butterflies and Urticaceae hosts (and Cannabaceae to a lesser degree, Fig. S4). All other host

families have been colonized in the last 15 Myr, after the divergence of the two largest clades 313 within Nymphalini, Vanessa and Nymphalis + Polygonia. Most species within Vanessa, both 314 extant and ancestral, are specialists on Urticaceae. V. virginiensis and V. cardui are the only 315 extant species that use more than two host families, and these hosts have likely been colonized by 316 their most recent common ancestor (node 38 in Fig. 8). On the other hand, the variation in host 317 use in the Nymphalis + Polygonia clade seems to be the result of host colonizations by multiple 318 species along the diversification of the clade. For example, in Fig. 8 we can see the colonization of 319 potential hosts by the ancestor of P. c-album and P. faunus (node 53) as well as strong 320 specialization on a new host by Kaniska canace. 321



Figure 8: Host repertoires at selected nodes of the Nymphalini tree (arrows in Fig. 7). Numbers indicate the node index (compatible with Fig. S3). For the only terminal taxa depicted, *Kaniska canace*, the observed host repertoire is shown. For all other repertoires, the posterior probabilities for states 1 and 2 are shown.

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DISCUSSION

The method we develop here to infer the evolutionary history of host-parasite associations has many advantages over previous approaches. First, it is based on stochastic models and on established principles of statistical inference, which means that it provides a robust framework for

characterizing the evolutionary processes that shape host-parasite associations and for selecting 326 among alternative coevolutionary models. Second, our model introduces the novel concept of a 327 host repertoire, which we think is an important step forward. Besides accounting for the 328 possibility of parasites having more than one host over time scales of macroevolutionary 329 significance, we can now directly infer the influence of host relatedness and host traits on the 330 process of gaining new hosts. Third, the stochastic model of host-parasite coevolution that we 331 introduce here is, to our knowledge, the first that explicitly accounts for evolution of the 332 fundamental host repertoire. By recognizing the fact that a parasite may have potential hosts in 333 addition to its actual hosts, and that the set of potential hosts may persist over time, the dynamic 334 of the model changes. What would otherwise have appeared as remarkable repeated patterns of 335 colonization of the same host lineages can now be explained as the effect of frequent transitions 336 between potential and actual hosts in an otherwise conserved host repertoire. 337

Our model can readily be extended in many interesting ways. The version we present here 338 accounts for the effect of host phylogeny by allowing the rate of host gain to depend on host 330 relatedness. For simplicity, we assumed that the number of available hosts and host relatedness 340 remain constant over geological time. This would be appropriate for a group of parasites that 341 radiated after the relevant host lineages had been formed, which is arguably the case for the 342 empirical example we chose. However, it should be relatively straightforward to extend our 343 framework to account for more complex dependencies on host phylogeny. For instance, the host 344 configurations could be modeled as changing over time, reflecting host cladogenesis. 345

Another interesting direction for future research would be to modify the particular ways in which hosts and parasites coevolve. We note, for example, that Fig. 7 shows that host repertoires of *Vanessa* species overlap very little with the host repertoires of *Nymphalis* + *Polygonia* species, but it is not immediately clear what drives this pattern. One could design a model that allows the rates of host gain and loss to be influenced by evolving host traits — like secondary metabolites, growth form, or phenology, to mention a few examples relevant for insect-host plant associations — in addition to relatedness among hosts. Or, one might extend the model to allow closely

related parasite lineages to competitively exclude one another from host usage, similar to how competing lineages might exclude one another from geographical regions (Quintero and Landis 2019). Finally, one might introduce a biogeographical component to the coevolutionary process, requiring parasites to be in sympatry with their actual hosts, while allowing parasites to be in sympatry or allopatry with their potential hosts. Statistically comparing such model variants will help illuminate drivers of host-parasite co-evolution.

A potential concern with our approach is that already the basic version of the model is 359 fairly parameter-rich. Given the type and amount of data that we can likely collect on 360 host-parasite associations, is there enough statistical power to select among the models of interest? 363 And is it possible to infer the model parameters of interest with a reasonable degree of accuracy? 362 Overall, our results are encouraging in this respect. The simulations indicate that it is 363 possible to infer the true parameter values of the basic model regardless of the level of 364 phylogenetic conservatism in both parasites and hosts (Fig. 3). When the rates of colonization of 365 new hosts are strongly dependent on the phylogenetic relatedness of hosts, then we are also able 366 to distinguish between models with or without host relatedness effects using Bayes factors (Fig. 367 4). However, our ability to select the correct model decreases when the effect of host phylogenetic 368 relatedness is low ($\beta \leq 1$), that is, when models become more similar. Further studies will have to 369 show to what extent the sensitivity of the model test can be increased by selecting appropriate 370 priors and improving the sampling of parameter space close to the boundary condition satisfying 371 the restricted model. One option is to relax the assumption that β is non-negative, which would 372 simplify the sampling of values close to $\beta = 0$. It will also be important to explore how dataset 373 sizes and tree shapes, for both hosts and parasites, influence our ability to distinguish the models 374 when the effect of host phylogeny is small. 375

Importantly, the empirical analysis indicates that the method is able to model the evolution of fundamental and realized host repertoires even when the information about potential hosts is lacking. This significantly increases the applicability of our method, as information about fundamental host repertoires is missing for most host-parasite systems. Potential host data is

difficult to collect, as it requires experimental testing of a large number of potential host-parasite 380 pairs. A possible improvement of our method, which we did not explore here, would be to model 381 uncertainty in the observations of non-hosts when data on potential hosts are missing. That is, if 382 we had no information about a host species being used by a particular parasite, we would 383 translate that to a certain probability p of the species actually being a non-host, and a 384 complementary probability 1 - p of it being a potential host (Kuhner and McGill 2014). Modeling 385 this observational uncertainty could help reduce the bias in parameter estimates that we observed 386 when data on potential hosts were missing and all 0 states in the dataset were inappropriately 387 treated as true non-hosts. This extension would also allow us to make predictions about host use 388 abilities in extant parasites. These predictions could then inform experiments that aim to 389 characterize fundamental host repertoires. 390

We demonstrated the empirical application of our approach with a Bayesian inference of the coevolutionary history between 34 Nymphalini butterflies and 25 angiosperm families. We estimated the rate of host repertoire evolution along each branch of the butterfly tree as being between 0.33 and 0.93 events per million years. Bayes factors favored the independence model, where the probability of gaining a given hosts is not affected by the phylogenetic distance between hosts. As explained above, this does not necessarily mean that host relatedness plays no role, only that the effect is not large enough for us to detect it with the current approach and the given data.

Estimates of gain and loss rates were not symmetric, and the rates also varied between 398 states. According to our results, gain of the ability to use a host, λ_{01} , is very rare (0.5% to 1.9% 399 of overall rate), whereas loss is common (47% to 73% of overall rate). On the other hand, 400 transition rates between states 1 and 2 were more symmetric and gain is more common than loss 401 $(\lambda_{12} \text{ between } 15\% \text{ and } 39\%; \lambda_{21} \text{ between } 6\% \text{ and } 18\% \text{ of overall rate})$. These rate estimates 402 support the idea that the use of the same host lineage by multiple, phylogenetically widespread 403 butterfly lineages is more likely explained by recolonization of hosts that have been used in the 404 past (recurrence homoplasy), that is, by transitions between actual and potential hosts, rather 405 than by completely independent colonizations of the same host (Janz et al. 2001). Note that 406

alternative scenarios that have been proposed in the literature to explain the evolution of 407 Nymphalini host plant preferences, for instance by involving narrow ancestral host plant ranges 408 and repeated independent colonization events, are also allowed by our model, but they are 409 inferred to be much less likely than the conservative host repertoire scenario. Yet, because the 410 potential host state is exited at the highest rate, the rate estimates also suggest that parasites do 411 not retain their potential host relationships for prolonged periods of time. The moderate rates of 412 transitions between potential and actual host states and the high departure rate from the 413 potential host state together help explain why phylogenetic "pulses" of recurrent host acquisition 414 manifest in some lineages but not others. 415

For example, the use of Grossulariaceae by two non-sister clades within *Polygonia* is best 416 explained by a scenario where Grossulariaceae was a potential host for the ancestral species (node 417 60 in Fig. 8) and was subsequently gained as an actual host twice (at nodes 53 and 58, Fig. S4). 418 The ability to use Salicaceae host plants seems to be even older. Salicaceae was likely a potential 419 host for the ancestor of Nymphalis + Polygonia and later became an actual host in three different 420 parts of the clade. If potential hosts were not explicitly modeled here, these transitions would 421 look like three independent colonizations of a plant group that is very distant from the ancestral 422 host (Salicaceae and Urticaceae diverged about 90 Ma). Instead, we could show that what might 423 appear as big and sudden host shifts, are in fact the result of retention of ancestral host use 424 abilities. 425

⁴²⁶ Understanding how ecological interactions change is crucial if we want to predict both ⁴²⁷ short and long-term consequences of global mixing of biota (Hoberg and Brooks 2015).

Host-parasite interactions are of particular interest given the risk of emerging diseases, which can affect human populations directly and indirectly through their effects on crop species and wildlife (Brooks et al. 2014). Our method was designed to quantify changes in host-parasite associations by modeling the process of gaining and losing hosts, thus allowing us to make predictions based on host-parasite history. Hopefully, our approach will not only generate deeper insights into the evolutionary dynamics of host-parasite associations but also help humankind mitigate some of the

⁴³⁴ risks incurred by current environmental change.

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